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FROM CATEGORIES TO CONTINUA FOR COMPOSITION AND IMPROVISATION:

Manipulating Distributions of Pitch Intervals in Prime-number Tuning Systems

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Purposes and Introduction

In recent work (Dean and Evans 2024), I have argued for the potential creative utility of sequentially modifying categorical musical structures towards continua, exploring intermediates as expressive vehicles. The main purpose of the present article is to illustrate ways to explore the possible relatives of a given tuning system, on such a path towards a complete pitch continuum, using the prime-number scales (Dean 2009) as exemplar (see section Exploration). I extend, through this exploration, the current set of categorical pitch structures used across cultures. The scales developed here challenge a composer or improviser to treat harmonic and melodic progressions in novel ways, triggered by the features discussed in what follows. The effects (i.e. affordances, implications) of these features may also suggest particular differential treatments of harmony and melody in different pitch zones (see section Discussion, Musical Examples and Conclusion).

The virtually uniform Western adoption of tuning systems based on octaves and equal temperaments with a single multiplicative frequency ratio interval between adjacent scale pitches has been queried (Will 1997; Dean, Bailes et al. 2008; Bongiovanni, Heald et al. 2023; Pushkar 2023). The main concern raised has been that octaves are perceived or generated quite imprecisely, even weakly (Wagner, Sturdy et al. 2022), yet they are the basis of dominant conceptions such as pitch class (see below). Pitch perception is complex, though perception of frequency differences is closer to a uniform multiplicative (i.e. logarithmic) approach than to an additive linear one (Burns 1999). But this does not exclude other such tuning approaches, such as those employed by many Asian and African cultures. Indeed, Pythagorean and Just tunings in the early history of Western music use numerous frequency multipliers rather than just one. Just tuning involves frequency ratios that are all integer, the largest within the repeating pattern being 2:1, the octave. At the opposite extreme, while continuously varying pitches cannot be played on a conventional piano, they are readily available on (unfretted) string instruments, many wind instruments (e.g. slide trombone) and on computational virtual multi-tuned pianos (Dean 2022). I emphasise, in this paper, pitch systems based on additive approaches, i.e. intervals based on frequency differences rather than ratios. This is not to argue that regular additive differences are preferable to regular multiplicative (logarithmic) differences but, rather, that they have been far less explored and hence deserve attention. Ideas of ratios in pitch are probably far more common because of the ratio nature of the harmonic series of frequencies but many sounds are inharmonic.

Let us recap the relevant terminology and background. The term nTET refers to equal tempered systems where every adjacent pair of pitches bears the same frequency ratio. If the system uses octaves (which have a frequency ratio 2.0), then n defines the number of steps in an octave (hence a uniform frequency ratio 1.06 for 12TET, a smaller value for 22TET, etc.). The pattern of n pitches repeats every octave ($1.06^{12} \sim 2.0$) and such octave-based systems can more precisely be termed nEDO (n equal divisions of the octave: features summarised in Figs. 1a–b). Pitch systems generally operate over more than 7 octaves. It is normal in music theory to consider the 12 EDO steps to be the same 12 pitch-classes (chroma) in all the different octaves, though perceptually this relationship is weak beyond one octave (Wagner, Sturdy et al. 2022). This weakness likely reflects several influences: the variability and imprecision of detection of the fundamental pitch of a note, the common occurrence of stretched octaves and the variability of spectral patterns of notes that are close to being one or more octaves apart (discussed in Burns (1999)). Every repeating pattern of pitch intervals (such as 12EDO) constitutes a tuning system that can occupy the whole audible range. Within the octave, different sequences of pitches can be chosen to constitute scale systems that also recur in each successive octave. A very few systems, discussed below, do not repeat over the audible range (Dean, 2009). The overall field of diverse tuning systems is often termed 'microtonality', with 'micro' indicating a particular interest in pitch intervals smaller than a conventional 12EDO semitone. But most commonly 'microtonality' is now used to indicate any pitch system that differs from 12EDO.

Milne and others have broadened the term 'dynamic tonality' i.e. systems open to live manipulation by a performer. While retaining a pitch span over which patterns of frequency ratios recur, they have introduced systems where more than one frequency ratio may be selected (Milne, Sethares et al. 2008; Milne and Prechtl 2008). For example, within their *Viking* synthesiser software, possible ratios drawn from a continuum can be chosen to determine, first, the span over which the pattern repeats: usually octaves, stretched octaves, tritaves (frequency ratio 3:2) or quintaves (5:2). *Viking*, secondly, allows separate generator ratios to be set, that usually determine a pitch interval adjacent to the EDO perfect fifth (1.06^7), but sometimes adjacent to the minor or major third (1.06^3 or 1.06^4). The remaining members of the repeating pitch ratio set are determined consequent upon this, so, with a generator frequency multiplier of n and a starting pitch of f, the first generated pitch is nf, the second n^2f, and the n-th n^nf. Once the frequency exceeds 2f, it is brought back into the starting f-2f octave (by removing the appropriate number of octaves from its frequency). This results in scale sets with varied ratios between adjacent pitches, potentially creating a continuum of both scales and tunings that are usually not ET but that still depend on the chosen frequency ratio. Adjustments are often used to allow the scale pattern to include octaves.

In contrast to these systems based on repeated frequency ratios, I previously created a totally unequally tempered family of prime-number scales, with 81 or 91 steps, that avoids repeating frequency ratio intervals but instead emphasises frequency difference intervals, several of which recur in different frequency ranges. These are now included in the Scala database of scale systems (Dean 2009). Having chosen an (unsounded) base frequency for the system, each member is created by successively multiplying it by the prime numbers (2, 3, 5, 7, 11, 13, 17,…), forming a series of unique ratio intervals. Prime numbers are those only divisible by one and themselves. Since the base pitch is not included, there are no octaves: if n is a prime number, 2n cannot be prime by definition. Because of the disposition of prime numbers, there are no repeated ratio-intervals (though several prime numbers are separated by a difference of 2 or 3, creating recurrent frequency differences). A scale structure devised within this system needs to cover the whole audible frequency range (because of the lack of frequency interval and hence octave repetition). These systems have been useful in some of my compositions, and have been subject to limited perceptual study (Leung and Dean 2018a; Leung and Dean 2018b, Leung and Dean 2018c), showing that, for most people (musically trained or not), it is more difficult in short-term experiments to learn pitch interval patterns (melodies) in this unfamiliar system than in several (familiar) equal tempered systems. Again, this is no reason for discounting its expressive possibilities, especially for avid repeat listeners.

As Fig. 1 shows, the 81-primes system, though diverse, is quite biased in its distribution of frequency differences and ratios. I develop, in this paper, approaches to manipulate such frequency distributions and indicate how they point towards uniform additive steps and ultimately to complete continua (with any pitch being possible). I suggest that the intermediaries are potentially interesting tuning systems for music-making.

I considered two main factors. First, there seems limited point in having discrete pitches that are so close in frequency that they cannot be distinguished within a melody (but see further comment below concerning chords comprising such pairs of pitches). The Just Noticeable Difference (JND) for pitch frequencies in simple timbres is generally around 1Hz (1 cycle/second) in the frequency range 10–3,000Hz (Schneider 2018; Arndt, Schlemmer et al. 2020). As frequencies increase towards the routinely used audible maximum for pitch fundamentals (around 4,100Hz), the JND increases progressively to about 7Hz. So, one can reasonably devise systems with step size at or above these JNDs. Note that both pitched and unpitched sounds contain important perceptible higher frequences up to around 22,050Hz (the effective frequency maximum of conventional CD-quality audio). Second, I considered the number of steps within the 20–4,000Hz range in the tuning systems of the Scala database: there are few with scale structures containing more than the 81- (or 91-) primes scales (Dean 2009). Most Scala scales, moreover, repeat seven times across the whole audible range of the corresponding tuning system (unlike the primes scales). So, conventional 12-, 22- or 31-EDO tuning systems would contain 84, 154 or 217 steps respectively. The rarer 53-EDO scale presents more steps in the complete system, while the Bohlen-Pierce system (most commonly equal tempered, and repeating at the tritave, comprising 13 pitches per tritave (Mathews, Pierce et al. 1988; Loui, Wessel et al. 2010)) has fewer (c~60) than the 12EDO. It seems that the optimal overall number of discrete pitches in the audible range should probably be no more than 220.

Exploration: Modifying Prime Number and Related Scales by Inversions, Combinations and Randomisations

The principle of converting categorical tuning systems via a series of steps gradually towards complete continua requires identifying the features of the systems that should be perturbed and then doing so methodically. The composer or improviser can, after this, use their own processes of music creation to evaluate the utility (initially, solely from their own perspective) of the different systems they have made. Besides pitch height (the absolute frequency of a pitch) per se, the salient features of the systems developed are the frequency ratio(s) (multiplicative) and the frequency difference(s) (additive) that each pitch bears to its neighbours. There is interest in assessing variations of each of these features, since Western 12EDO tuning shows complete stasis in ratio interval at ~1.06 (Fig. 1b), and consequently, a progressive linear increase with frequency of the frequency differences (Fig. 1a) to a maximum of about 250Hz.

The 81-primes system (or scale) (Fig. 1c–d), in contrast, shows variable frequency differences (10–140 Hz). While there is some linear trend, the relationship is even better represented by a Loess (multi-segment) fit. There are repeated frequency differences, but no repeated frequency ratios. Correspondingly, the pitch step frequency ratios (range 1.01 to 1.67) show a weak linearly descending trend, much better represented by a Loess fit. In both respects (difference and ratio), the 81-primes scale is radically different from 12- (or other) EDOs.

To vary maximally both key features (difference and ratio), it would be desirable to consider both inverses of the 81-primes feature distributions and more repetitious distributions. The first can be achieved by constructing a system like the 81-primes in choosing an arbitrary high starting pitch (4,096Hz) and altering it successively by subtracting the result of multiplying the inverses of primes in increasing magnitude (i.e. starting with (1/largest prime) and continuing to (1/smallest prime)) by a chosen constant frequency. The result of this is shown in Fig. 2a–b, using frequency multiplier 2,640Hz to fit outcomes into the desired range. Now frequency differences range from 6 to 866.67 while ratios are from 1.001 to 36.91, in both cases substantially expanded. There is a descending trend in differences in relation to frequency, as intended, but the large interval between the first and second pitches (not shown) makes the scale of use mainly in the high register. As with the original 81-primes tunings, there are no duplicate ratio intervals, though sizes become very similar at high frequency. Unlike both 12TET and the original 81-primes scale, this system shows similar descents with frequency for both difference and ratio measures.

In moving to diverge further from the standard 12EDO system and produce more diverse distributions, I combined both primes pitch sets, removing the upper of any adjacent pair of pitches that were not at least 7Hz apart (taking account of the JND criterion already described), giving the Composite scale, Figs. 2c–d. This scale contains 143 members (within the target maximum), frequency differences 7.08–240 and ratios 1.002–1.667. Because of the addition of several steps between 2,000 and 4,000, the Loess plot of differences is now slightly biphasic (though the overall linear descent remains). The ratios plot primarily shows the original 81-primes variability and descent with frequency. One further step in this direction was assayed: adding to the Composite scale another primes system with root 30Hz instead of

Figure 1. Vertical lines (1a) indicate octaves, where pitch classes repeat. Red and green lines (c, d) are Loess multi-segment fits, unsegmented black (in all subfigures) is the linear fit.

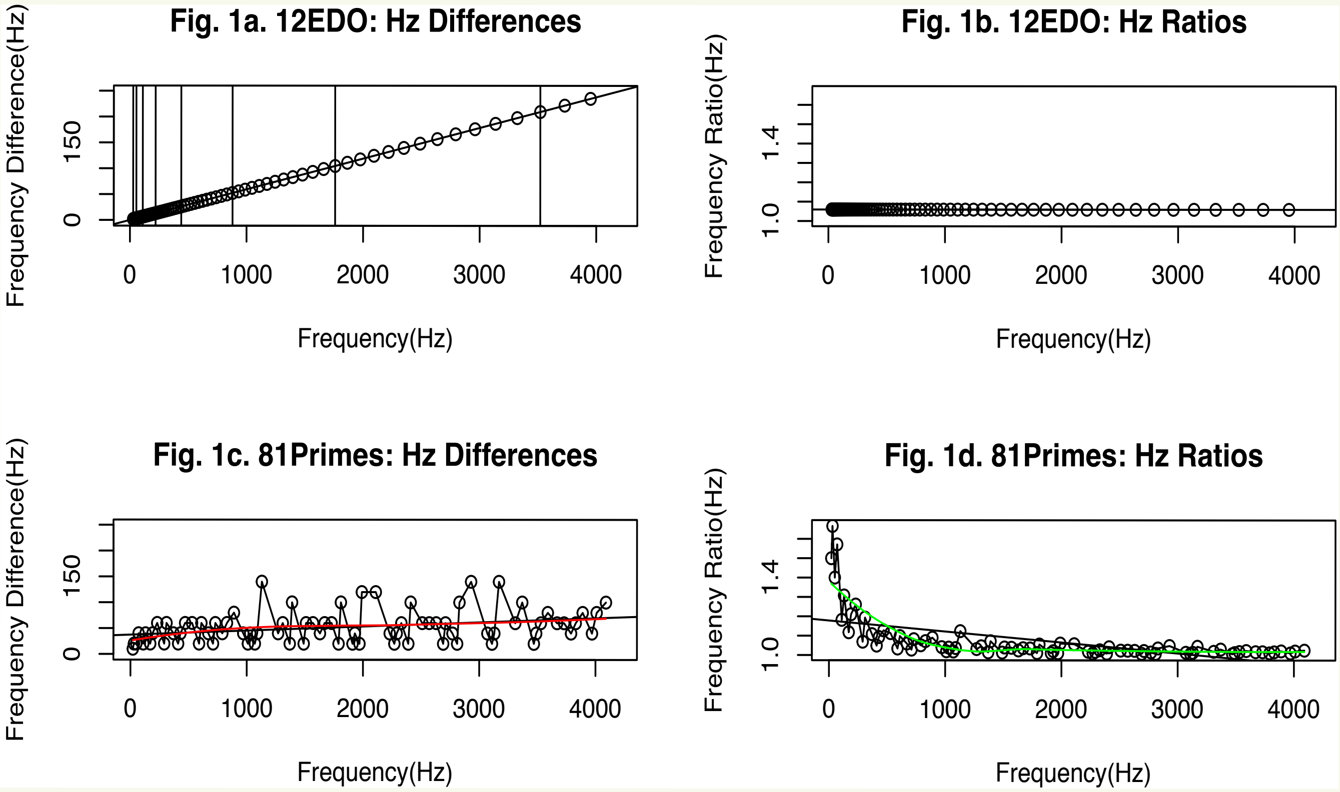
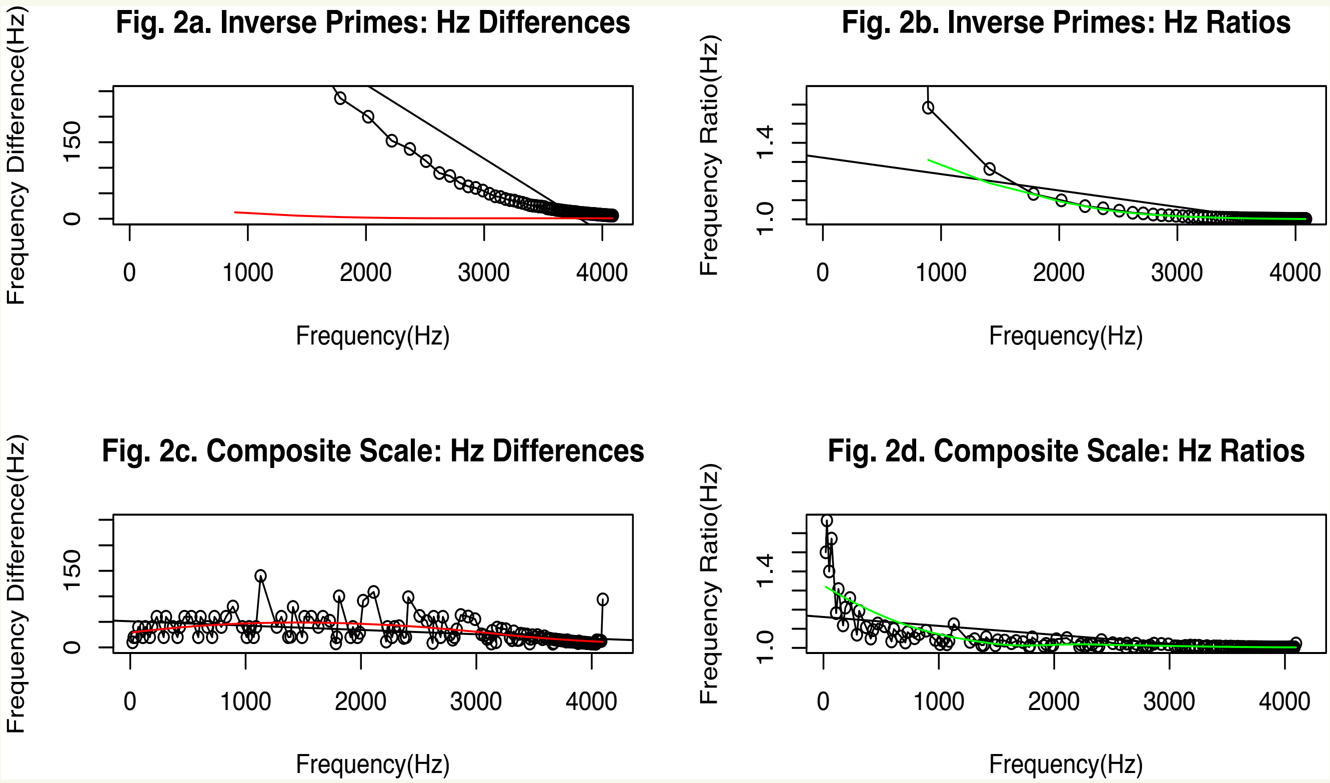


Figure 2. Red and green lines are again Loess fits. In 2a–b the lowest frequency member is not shown (for reasons of scale comparability) and the Loess fits are shortened correspondingly.



the earlier 10Hz, giving recurrent difference intervals of 20Hz, some of which were then removed because of the application of the >7Hz difference rule. This system (named Composite2) comprises 171 members.

Additional scales with randomised or uniform pitch difference steps can readily be obtained within the operative 20–4,100Hz range. For example, with ~205 steps all at 20Hz difference, every ratio differs and they decrease as frequency rises, as in Fig. 1d, and Figs. 2b, 2d. Alternatively, one might use ~ 205 at difference values randomised as 20Hz +/- random(10) with adjustments if necessary to avoid steps <7Hz and to cover the whole desired range (ratios again tend to decrease with frequency). This is shown as Random20 in Table 1. Another interesting approach relevant here (not shown) would be to use the Gaussian approximation to the prime number series, where the n-th prime is estimated as $n\log_e n$ (where $\log_e$ indicates the natural logarithm). The history of approximations to the prime number series (or instead to counts of prime numbers up to a certain range) has been well described (Zagier 1977). After a maximum resolution of about 400 additive steps at an average around 10Hz, too many intervals come close to the JND and are less useful. Another interesting computational possibility (also available as a switch on the PianoTeq physical synthesis instrument) is to invert the keyboard, so that the pitch pattern appears downwards from the highest frequencies rather than in its normal mode. At this point the next step is the complete continuum such as I have developed for the virtual piano (Dean 2022). Here the performer can choose how close in pitch to play (using a touch-sensitive interface such as an iPad) and judge acoustic effects of combinations closer than the JND, some of which are at the least timbrally interesting.

The Coefficient of Variation (CV) measures the ratio of Standard Deviation (SD) to Mean, an index of the variability of the values. As noted in the text, the removal of pitches that provided differences <7Hz from their predecessor was only done for the two Composite systems. With the inverse primes, 15 successive differences in the very highest register were between 6–8Hz (and were retained). With the Random20 system, the first implementation covering the whole pitch range up to 4,096Hz (actual maximum 4,098Hz) did not contain any differences <7Hz.

Table 1 shows some of the statistical changes in distributional pattern amongst the different systems described. Some of the salient features are the very low Minimum Differences (in the very low register) of 12EDO, though all are above the relevant JND. The other systems all have minima between 6–10Hz. System 12EDO, similarly, has an unusually large Maximum Difference (235 Hz) but one of the prime number systems exceeds this, while, as desired, the Random20 system is consistently small in difference step. Consequently, it is

Table 1. Statistical Features of the Pitch Difference Distributions of the Tuning Systems. The table headings indicate the following: nPitches is their number in the system. Min., Max., Med., Mean., S.D., CV. are respectively the Minimum, Maximum, Median, Mean, Standard Deviation and Coefficient of Variation of the pitch differences.

System	nPitches	MinHzDiff	MaxHzDiff	MedHzDiff	MeanHzDiff	S.D.HzDiff	CV
12EDO	88	1.64	234.94	19.6	47.80	59.76	1.25
81Primes	81	10	140	40	52.13	31.25	0.60
Inverse81 Primes	81	6.21	866.67	14.44	50.90	21.05	2.38
Composite	143	7.08	140	20	29.37	24.18	0.82
Composite2	171	7.08	100	20	24.53	17.70	0.72
Random20	206	10	30	19	19.89	6.01	0.30

Inverse Primes > 12EDO that have the largest CV. System 12EDO shows the largest SD for HzDiff, as expected from its multiplicative origin, while the other constructed systems quite well represent the possible downward range (to a minimum SD of 6 in the Random20 system). Another criterion that might be applied to choosing the system of interval ratios (depending on user or ideological aesthetic preference) is that of approximation to small integer ratios known as 'just' or 'pure', a result often termed 'optimal' (for a stimulating mathematical article on this, see Goldstein (1977)). Other generalised approaches, assuming 12 subdivisions of an octave and with controlled degrees of tempering, that is, deviation from just intervals, have been discussed (e.g. Farup 2014).

Discussion, Musical Examples and Conclusion

As mentioned, one purpose of the scales developed here is to encourage the treatment of harmonic and melodic progressions in novel ways, triggered by the scale structures, the musician's responses to them and the inclusion of possible distinct handling of different zones of the keyboard or pitch areas. For example, with the composite scale, the zone of frequencies greater than 2,500Hz, with numerous small intervals around 10Hz, can be used differently from the lower keyboard (with intervals >20Hz) in ways that do not apply with 12-EDO. For example, enunciating a given melodic or harmonic succession within a range of perhaps 12 adjacent pitches in the >2500Hz zone will be very different from doing so in the lower keyboard, and not just in average pitch height but also in cluster effects and in the degree of detectability of pitch contours. Combining different tunings on different keyboards is also effective, as major effects (such as beating) of inter-tone spectra work even dichotically (when delivered to separate ears), as illustrated in Sound Sample 2. As implied already, a pair of pitches that cannot be distinguished reliably when played successively, when played together will nevertheless almost certainly offer timbres distinguishable from those of either member alone. So, while perceptibility is necessary for utility of a tuning system, melodic separability of each of the scale note set from each other may not be. This view is also supported in Goldstein's discussion of temperament 'optimality' (Goldstein 1977).

In Dean and Evans (2024), we argue that familiarity and fluidity with a range of tuning systems can also help intercultural performers to work across musics with different traditions of tuning systems. Here I suggest that, on most instruments, listening to and thinking about the spectral combinations that arise from simultaneous multiple pitches in a tuning system can aid a performer in the development of sensitivity to expressive techniques on their instrument (some appropriately termed 'extended techniques'), whatever the music they interpret and whatever the instrument they play.

It is in this spirit that I approached the development and use of my 81-primes scale for the two sound examples attached to this article. Sound Sample 1 is an extract from *Ubasuteyama* (2008) and is about the ancient Japanese practice of Ubasuteyama [grandmother throwing mountain] whereby an elderly relative was taken up a mountain and left to die. The practice was the basis of a film by Shohei Imamura, *The Ballad of Narayama*. The text of the piece, by Hazel Smith, also refers to a Buddhist allegory, designed to illustrate self-forgetfulness and concern for others, about an old woman who was taken up the mountain and who scattered twigs to help her son find his way back. The piece was conceived as a compositional structure, with improvised material. The original 81-primes scale is used in the form of noise-bearing synthetic electronic sounds made in MaxMSP (a leading algorithmic music coding and performance platform expressed in graphic objects), with just detectable spectral centroids (which the performer at least can consider their pitches). The scale is also used on a synthesiser that creates continuously sustained keyboard instrument sounds with precisely detectable pitches. The synthesiser was set up with its MIDI notes defined as the pitches of the scale. The improvisation of some of the material led me to observe and exploit the fact that some synthesiser pitches or pitch combinations give somewhat unusual percussive effects in their transient attacks, while yet having a potentially long-sustained, non-decaying continuation (unlike most percussion instruments, which inevitably decay quite quickly).

Sound Sample 2 is *Stretches, Joins, Overlaps 1* (2024), created while writing this article and with the creative and cognitive issues I have mentioned strongly in mind. It was produced by first recording (in MIDI and audio) an improvisation using the new composite scale described above on a MIDI-keyboard that was sounding the commercial physical synthesis piano, PianoTeq, also used in earlier work on a continuous pitch virtual piano (Dean, 2022). PianoTeq has physical models of major brands of excellent grand pianos (e.g. such as Steinway and Yamaha). When playing isolated tones, it makes highly plausible piano-like sounds whatever the pitch pattern it enunciates, unlike sample-based electronic pianos where interpolation between samples to produce intervening pitches can be unconvincing. The physical models of PianoTeq have presumably not been extensively tested for acoustic precision with respect to sounds made by retuning genuine grand pianos and may or may not be highly accurate. In current versions of the software, there are switches to allow a user to somewhat control which physical mechanisms are extremely stretched in the model so as to generate the required pitch combinations. In any case, for a performer (or creator) the salient feature (as always) is to interrogate how melodic and harmonic pitch groupings sound and appeal and how they can be expressive.

I used a MaxMSP patch to send extended MIDI signals, with minute pitch differences expressed in the code, to PianoTeq. My improvisation exploited the fact that notes and particularly chords in the very low register again sounded very percussive and noisy, with a complex progressive decay. A few particular, mid-register chords had elements of this effect, while high notes and chords were much clearer and more pitch-bearing. Some combinations and registers could produce sounds that seemed plucked and not exactly as the grand piano sounds when plucked. These were amongst the features that I attempted to exploit in my improvising. I undertook two improvisations and chose the second for further use. At that point, following on earlier ideas (Dean and Evans, 2024), I conceived the idea of using again the same MIDI-setquence that I had just recorded alongside audio, but played as normal 12EDO pitches and in reverse order. Thus, for example, a high-resolution note of MIDI value 60.79 would become 60.0, and be heard as middle C in the current 12EDO tuning chosen in PianoTeq (with C60=256 Hz) rather than what might have been a pitch at least 100Hz away in the composite system. Interestingly, with the PianoTeq settings in force (one can vary attack, nature of the instrument, etc.) some mid-register chords were again slightly more percussive than most. I made the final piece by a compositional interplay between chosen segments of the reversal of the audio of the original MIDI file (converted to 12-EDO) and the audio of its initial composite primes tuning version. Unlike reversing an audio file, reversing a MIDI-file produces the normal sequence of attack and decay for each event, but just in reverse order. Besides controlling overall dynamic changes and spatial distribution of sounds, I again took note of juxtapositions that enhanced what to me seemed appealing spectral effects between the expressed pitches.

I have chosen these sound examples partly because their interests for me go beyond the pitch pattern implications for melody and harmony of the chosen tuning systems and rather emphasise timbral and resonance features. But, to recall some of the earlier arguments, changing the nature of melody and harmony has been a concern of most tuning systems in the history of music throughout the world, and systematic approaches, such as I describe, may allow further productive discovery. I hope these initial observations can indicate some of the appeals of the exploratory pitch path I am describing.

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SOUND SAMPLE CREDITS

There are two items:

1. An extract of *Ubasuteyama* (2008) by Roger Dean (sound) and Hazel Smith (text, text performer), is performed by austraLYSIS. The piece is available on Wirripang CD (Wirr 11) and at www.australysis.com. It uses Dean's 81-primes scale in the keyboard and electroacoustic parts.
2. A new piece, *Stretches, Joins, Overlaps 1* (2024) for one composite primes-tuned PianoTeq grand piano and one 12EDO-tuned, has been made for this article by the author.

ABSTRACT

I have recently codified and analysed some general and potentially valuable approaches to finding novel musical vehicles for expression. One of these is the process of gradually converting any set of categorical elements of musical structure towards a continuum, allowing a musical creator to then make a personal assessment of the utility and expressive applicability of each gradation within their own work. The purpose of this article is to illustrate this with respect to pitch structures, starting from normal Western tuning, such as that of the piano, and moving towards continuous pitch structures, in this case emphasising intermediaries based on prime number systems, one from my earlier work and others created for the article. I provide pointers towards the perceptual and cognitive aspects of pitch structures, alongside the consideration of their utility for music composition and improvisation. I complement the argument with two concrete audio examples: an extract of an ensemble piece made using my original 81-primes scale in the keyboards and a complete keyboard piece made with some of the graded tuning intermediaries discussed here.

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Keywords. Pitch; tuning; scales; categorical pitch; continuous pitch; prime numbers; ratio intervals; additive intervals.

ABOUT THE AUTHOR

Roger Dean, composer, improviser and researcher in music science, directs the creative ensemble austraLYSIS and is a professor at the MARCS Institute for Brain, Behaviour and Development. With Luers (image) and Smith (text), he received the international Robert Coover Prize for Electronic Literature (2018). He has performed diversely as bassist, pianist and computer artist with the Academy of Ancient Music, Australian Chamber Orchestra, London Sinfonietta, Graham Collier Music, austraLYSIS and its predecessor in the U.K. (LYSIS), Bailey, Parker, Evans and Slater and many others. He has made ~70 commercial recordings (e.g. *Dualling* (by austraLYSIS, Earshift 085, 2024)) and numerous radio and multimedia pieces. He is widely cited for the book and model, *Practice-led Research, Research-led Practice in the Creative Arts* (Edinburgh, 2009). He was formerly a UK professor in biochemistry, led the Heart Research Institute in Sydney and then moved to lead the University of Canberra.

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